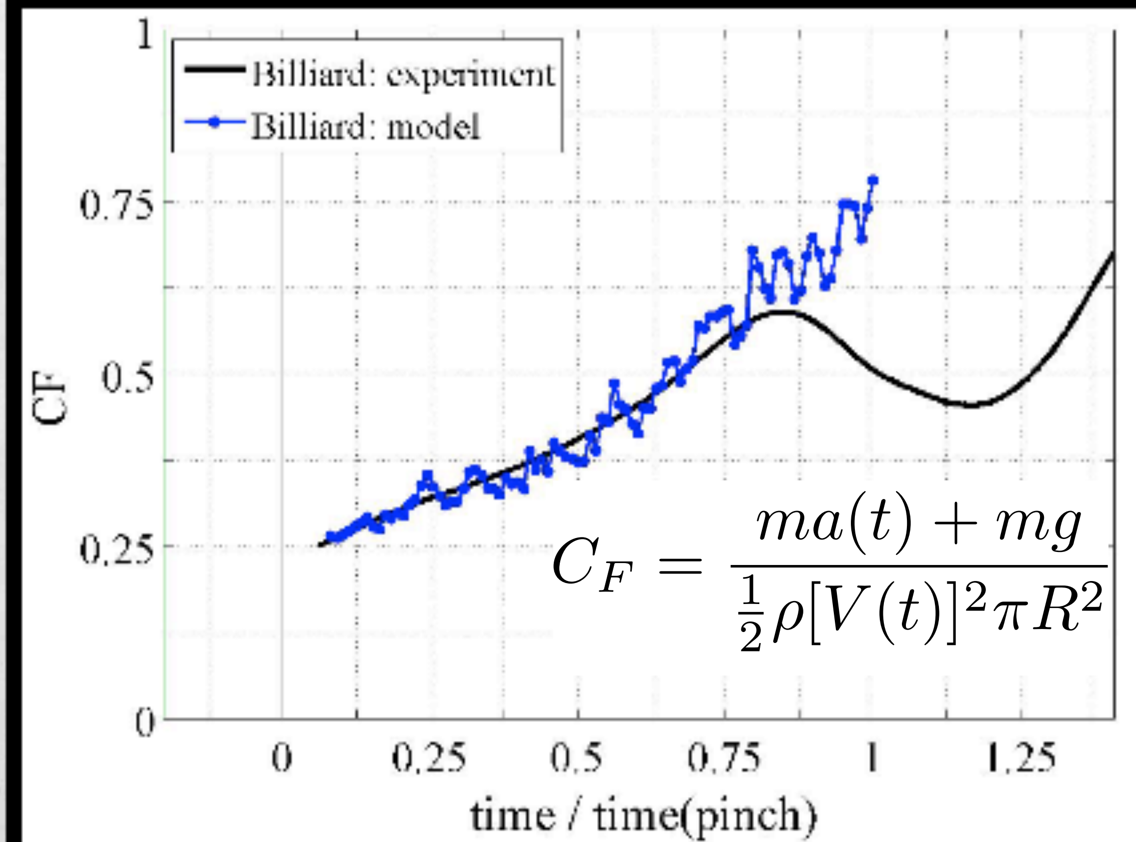
Evaluating derivatives of experimental data using smoothing splines



Brenden Epps,

Tadd Truscott,

Alexandra Techet

MIT
Department of Mechanical Engineering

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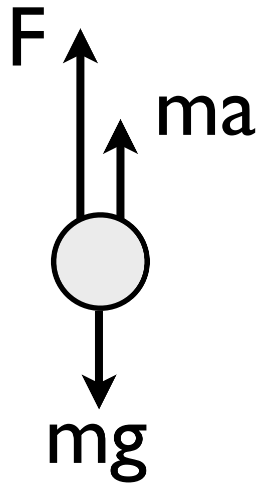
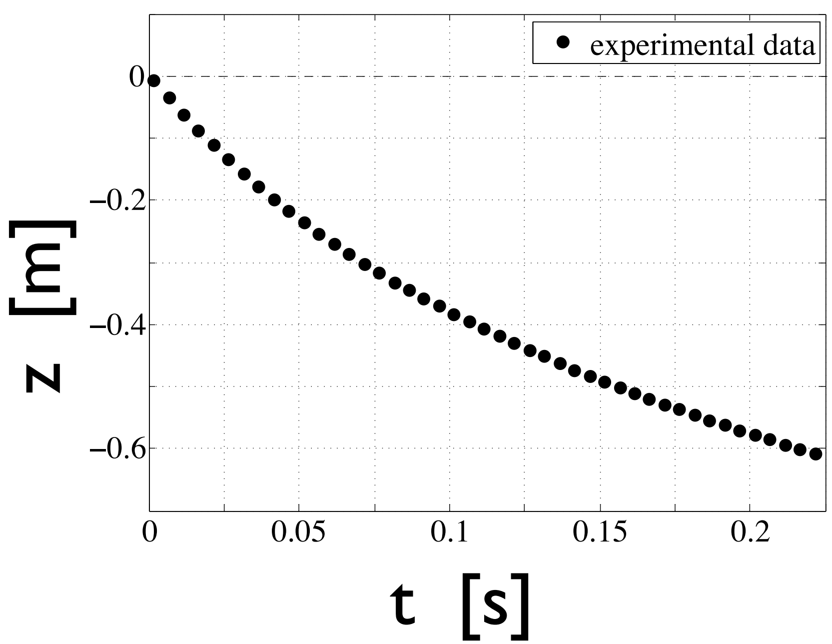
force coefficient



$$F(t) = ma(t) + mg$$

$$C_F(t) = \frac{F(t)}{\frac{1}{2}\rho[V(t)]^2 A}$$



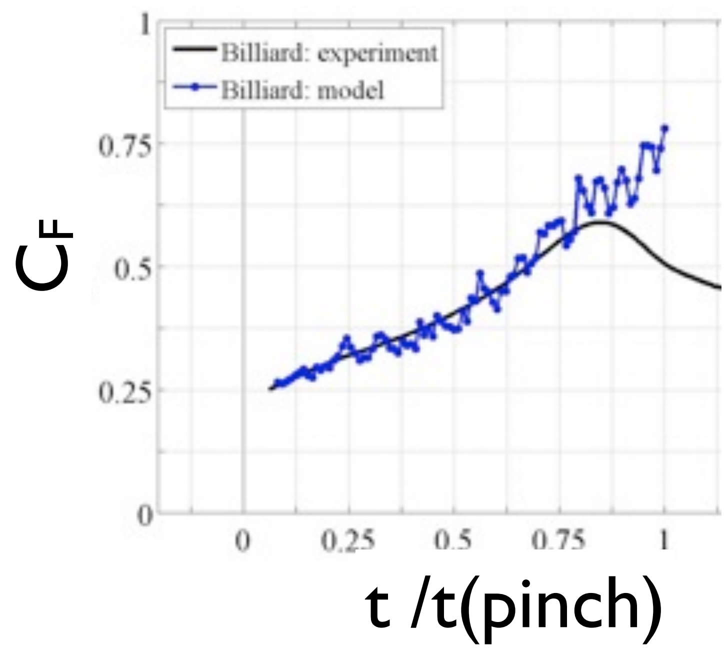
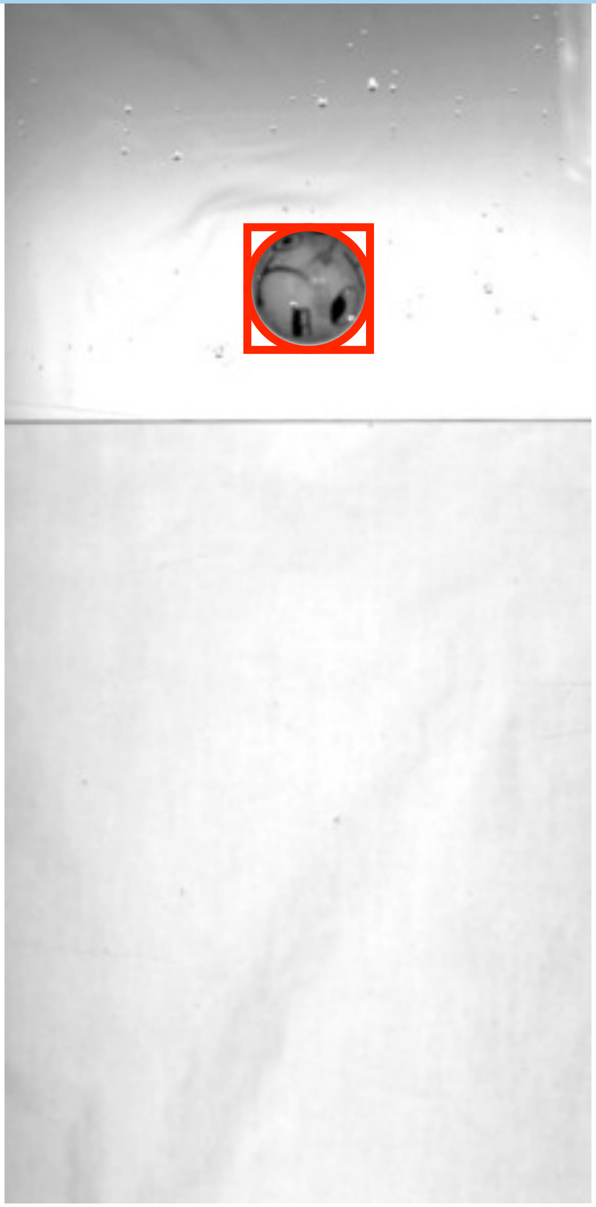
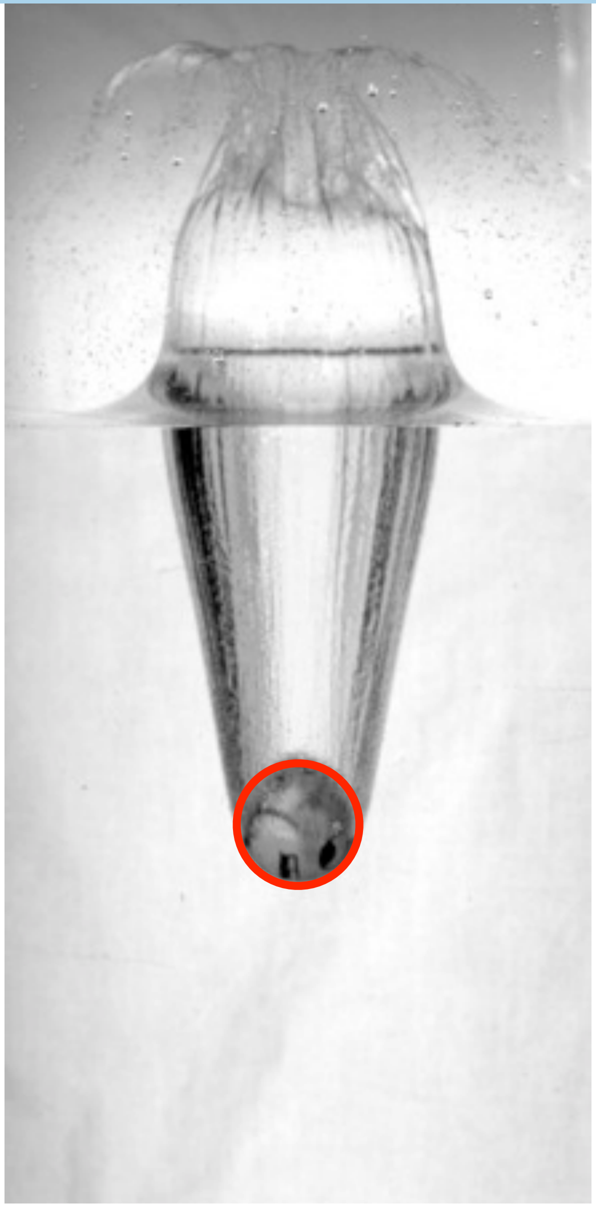
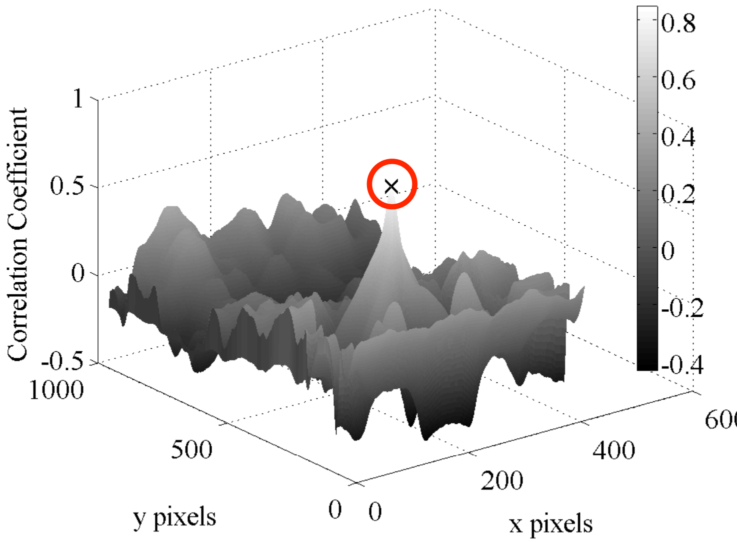
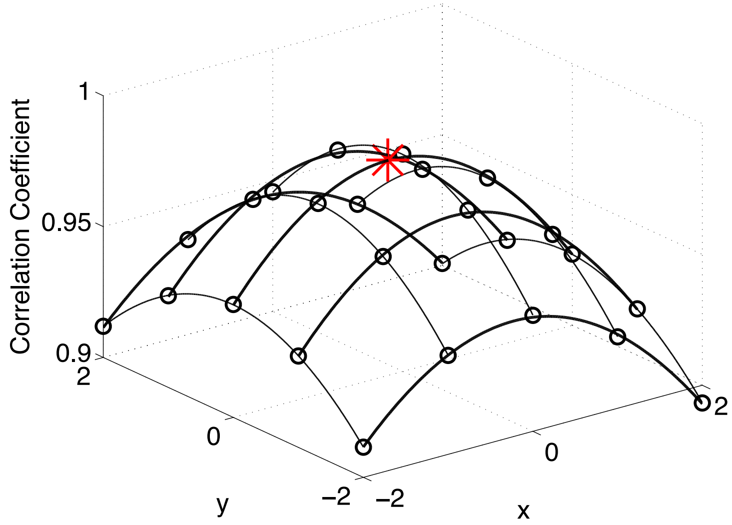


image processing



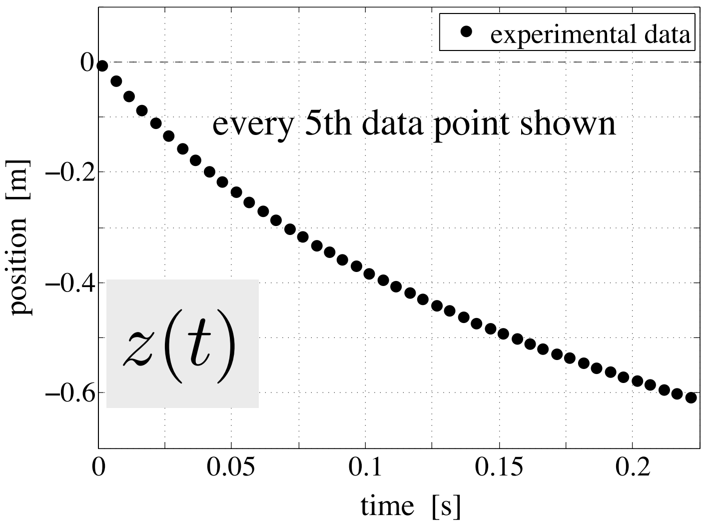


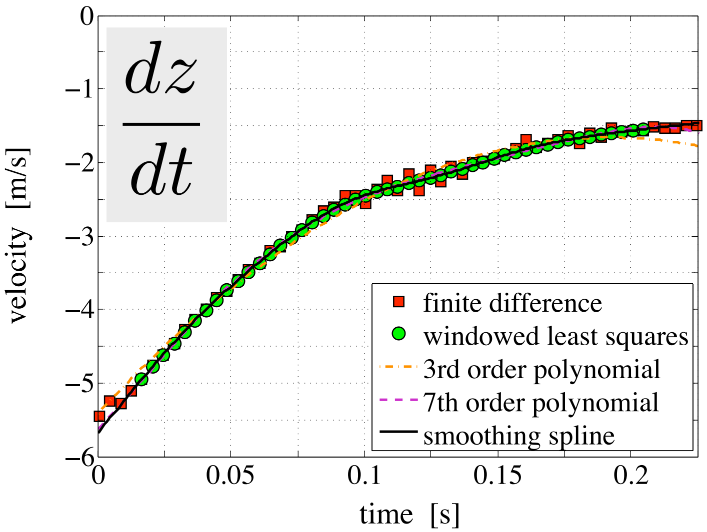


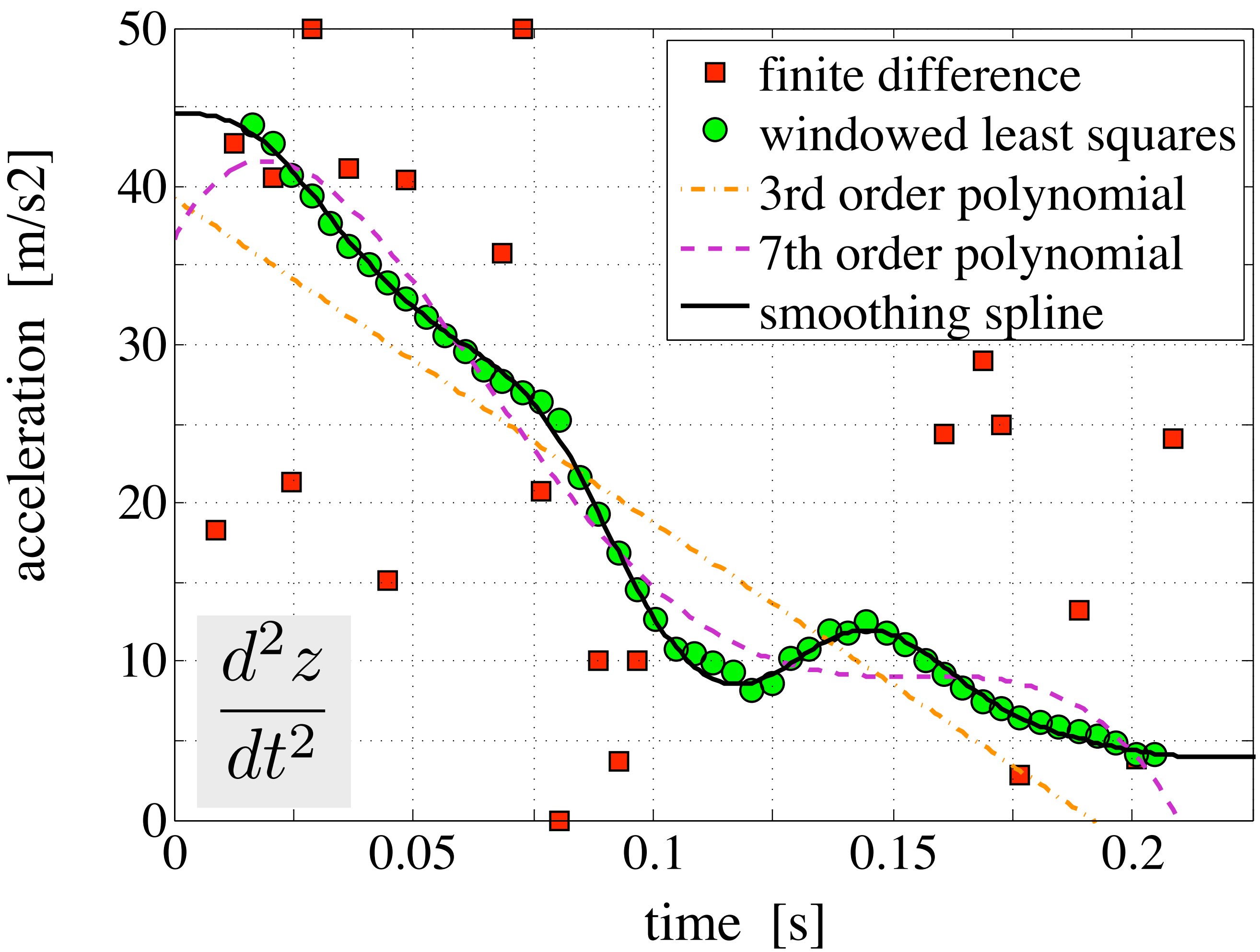


build template cross correlate find sub-pixel position

derivatives of noisy data

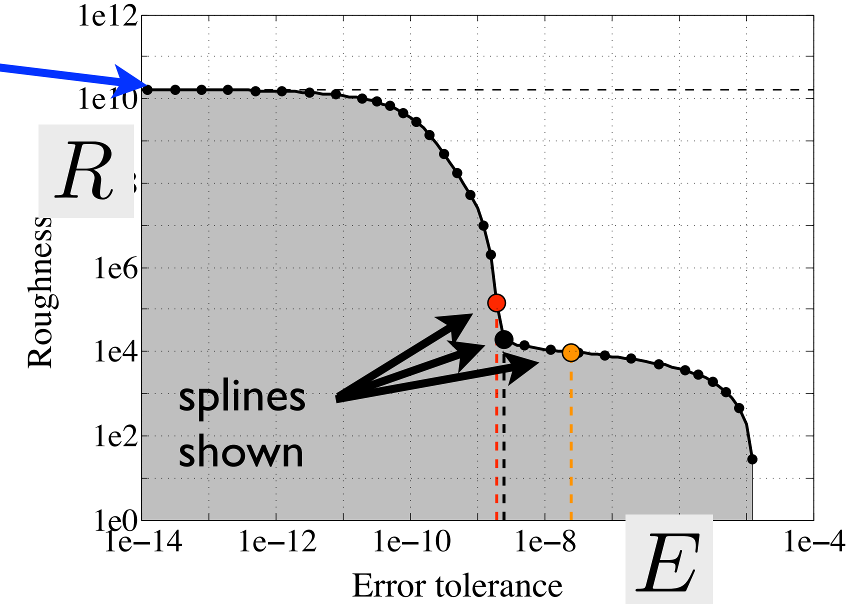






smoothing spline selection

Frontier of minimum-roughness splines



quintic spline: $s(t)$

error: $\bar{E}(s) = \sum_{i=1}^{N} |\tilde{z}_i - s(t_i)|^2 \triangle t$

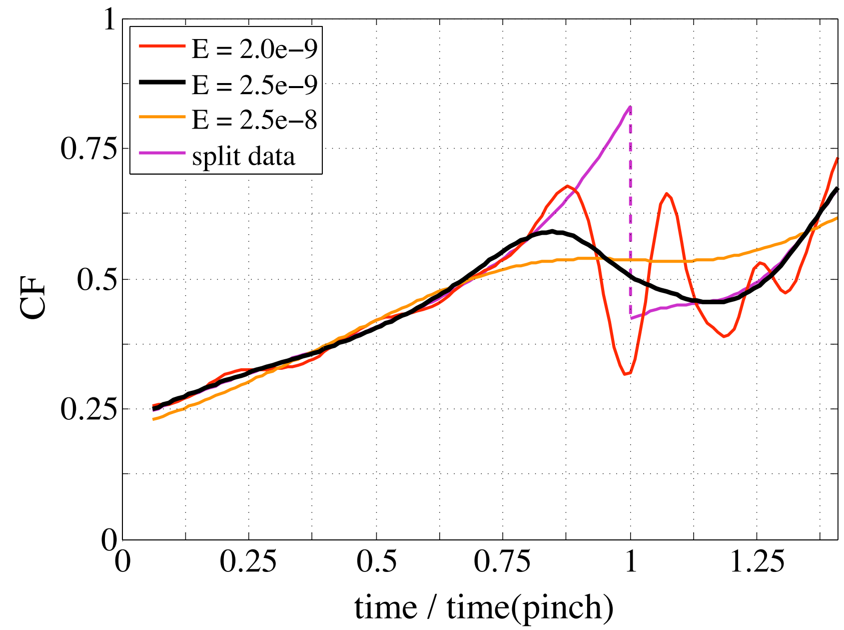
roughness: $R(s) = \int_{t_1}^{t_N} \left| \frac{d^3 s}{ds^3} \right|^2 dt$

choose: $E \equiv$ error tolerance

best fit spline for a given E: minimize $R(s)$ requiring $\bar{E}(s) \leq E$

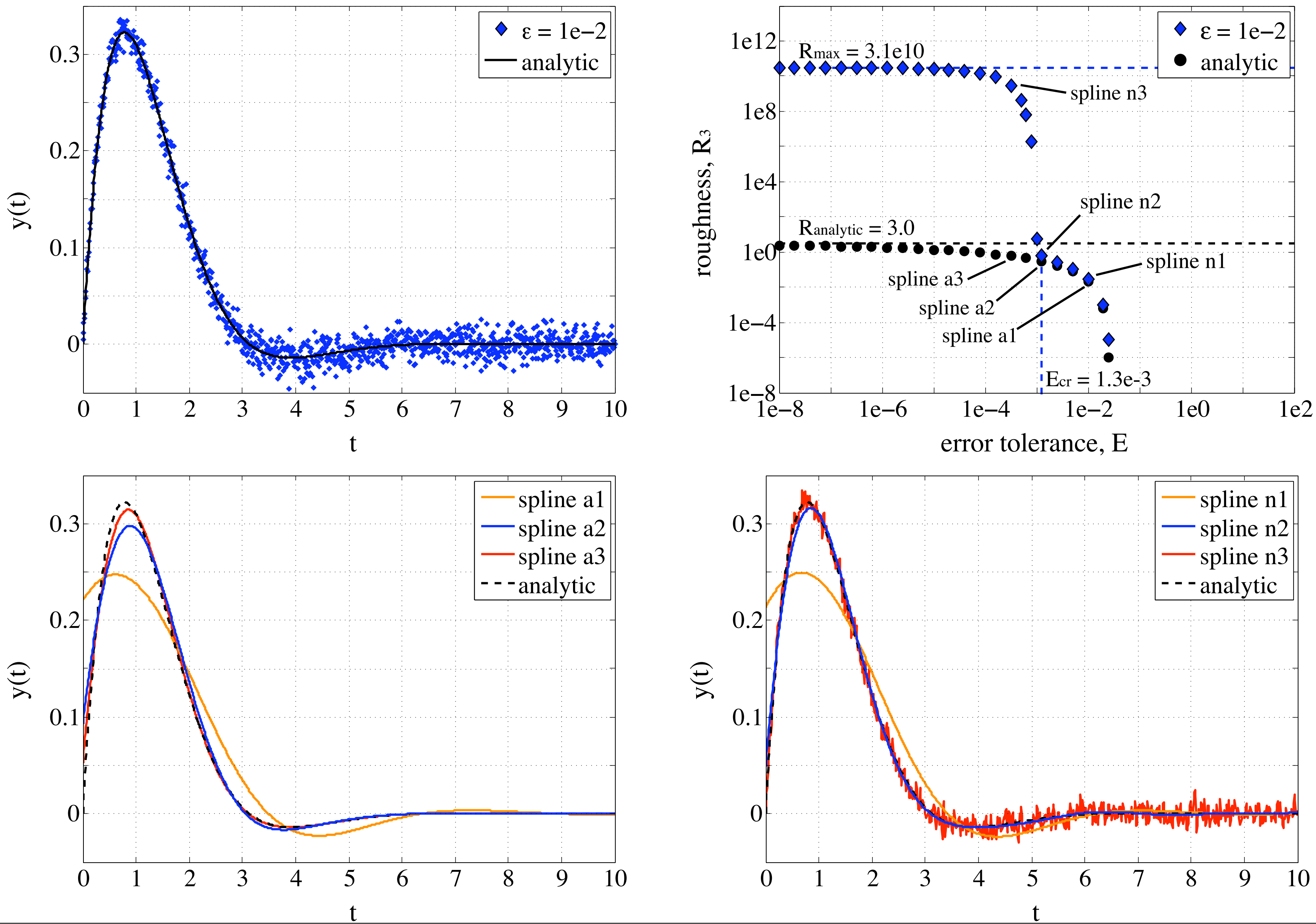
choose: $p \equiv$ smoothing parameter

best fit spline for a given p: minimize $p\bar{E}(s) + (1-p)R(s)$



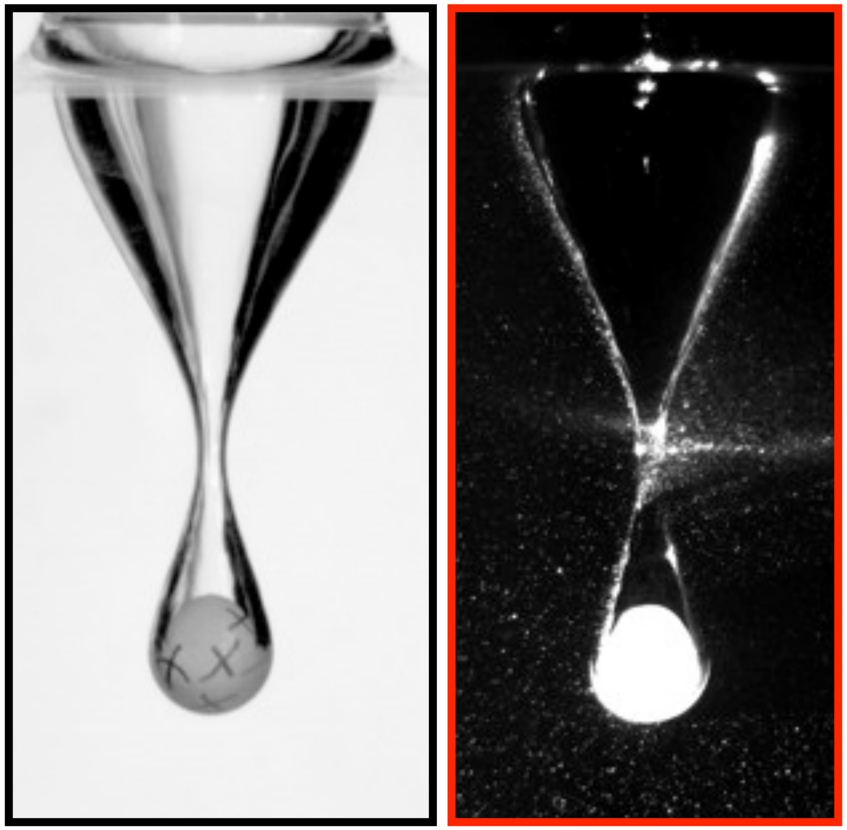
analytic example

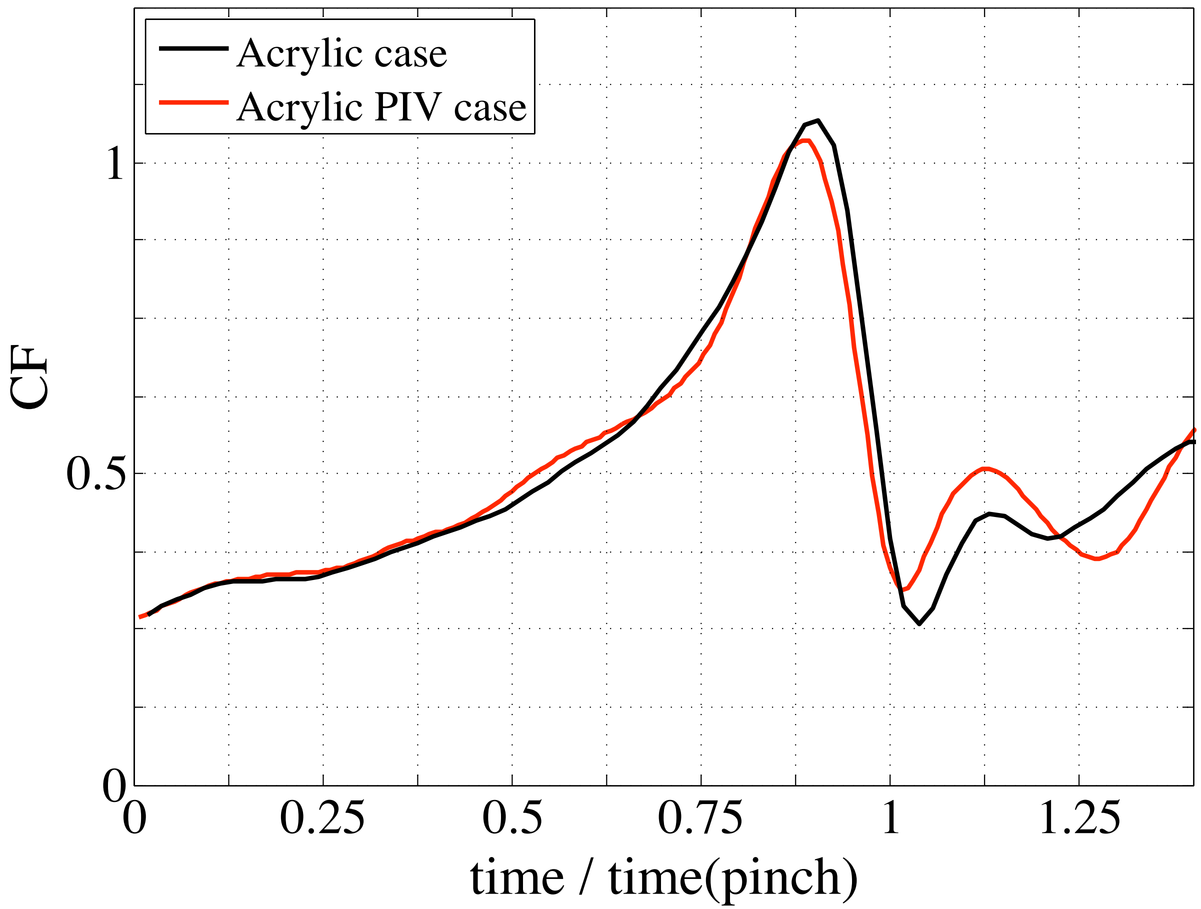
$$\tilde{y}(t) = e^{-t} \cdot \sin(t) + \mathcal{N}(0, \epsilon^2)$$



repeatability

Comparison of two cases
with acrylic spheres:





physical model

given experimental data:

$$H(t),\ V(t),\ R_c(x,t)$$

assume potential flow:

$$\phi = \phi_{\text{doublet}} + \phi_{\text{ring sources}} + \phi_{\text{point source}}$$

find source strengths (in sphere frame):

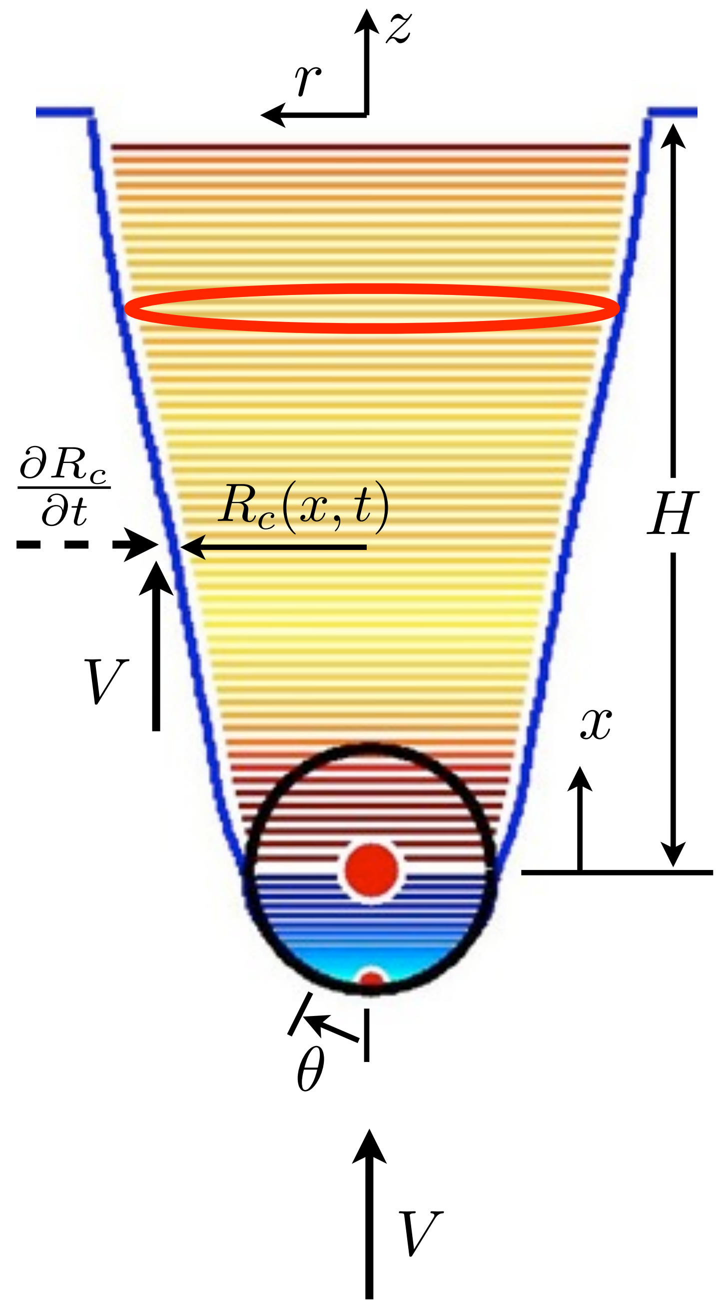
$$\vec{u}\cdot\hat{n} = \begin{cases} V\ cos(\theta) & \text{on sphere} \\ \frac{\partial R_c}{\partial t} + V\frac{\partial R_c}{\partial x} & \text{on cavity} \end{cases}$$

evaluate pressure on surface (in lab frame):

$$p - p_a = -\rho\frac{\partial\phi}{\partial t} - \frac{1}{2}\rho|\vec{u}|^2 - \rho g z$$

integrate to find force coefficients:

$$C_F = C_{F_u} + C_{F_b} + C_{F_h}$$



billiard ball case

billiard: $m^* = 1.8$

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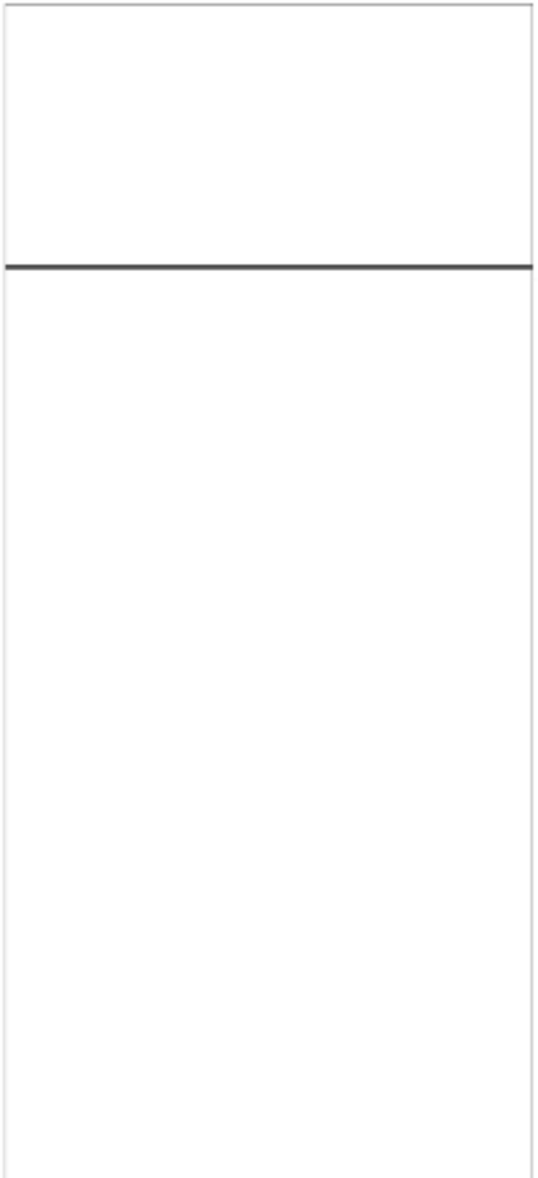
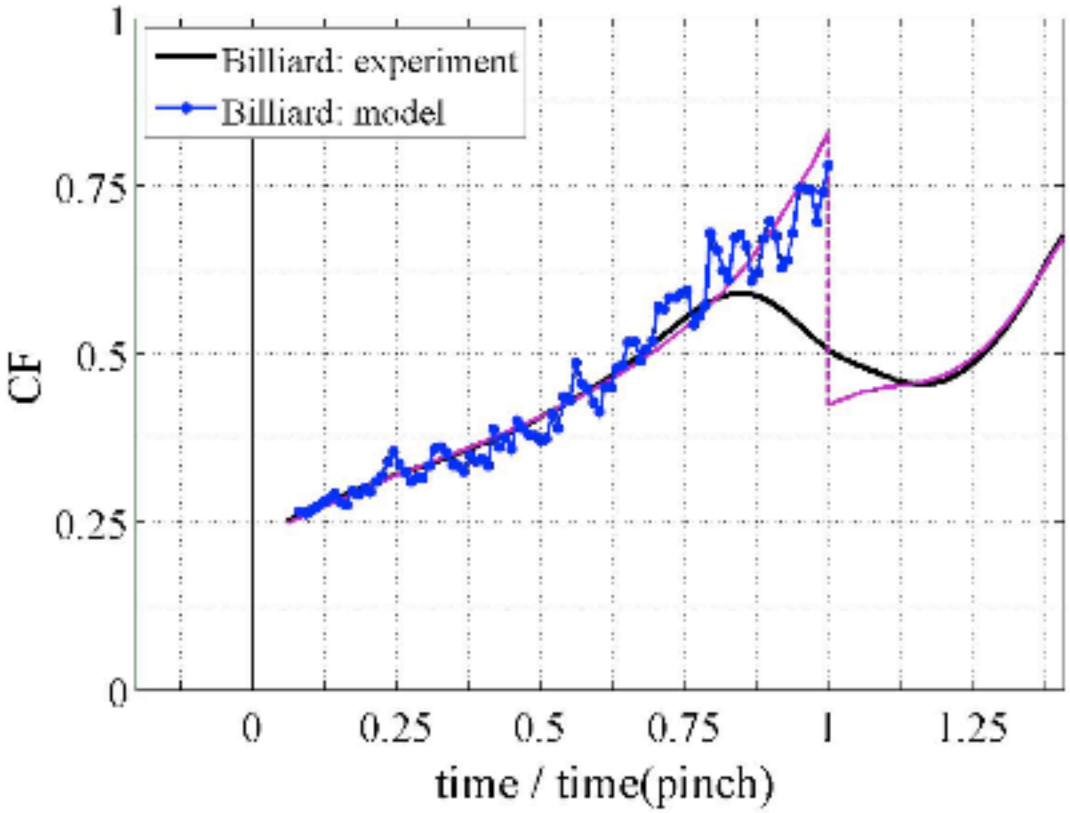


$$C_F = \frac{ma(t) + mg}{\frac{1}{2}\rho[V(t)]^2 \pi R^2}$$



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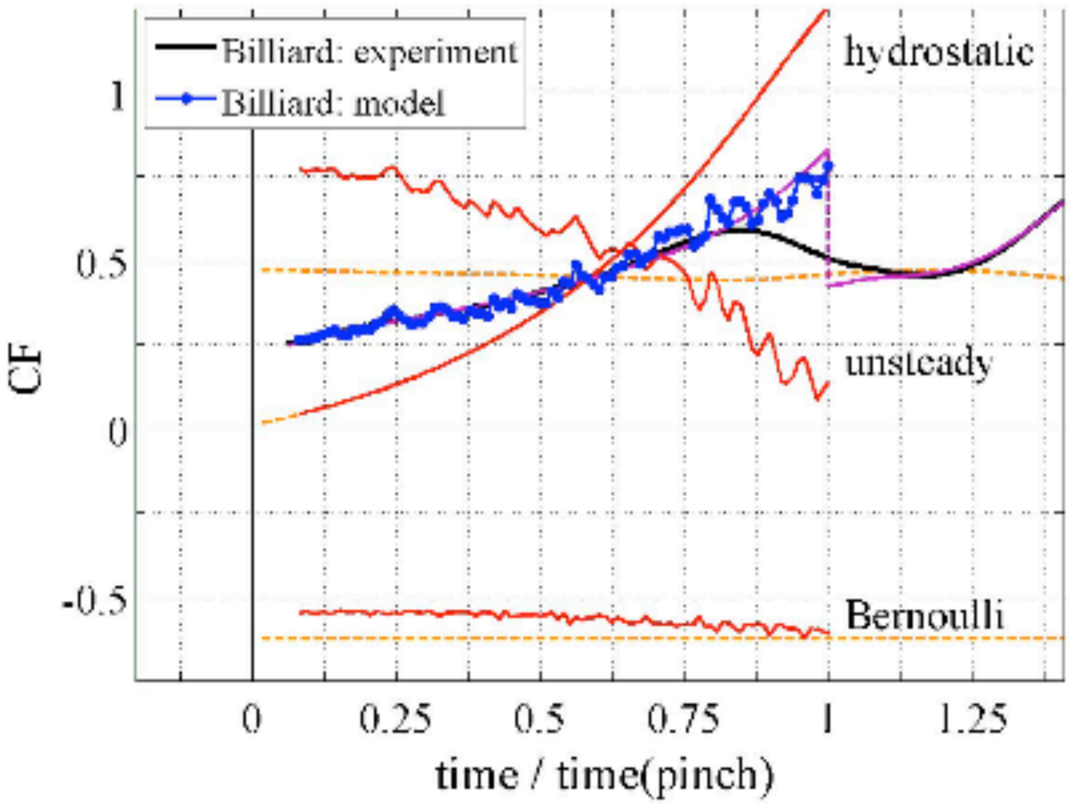
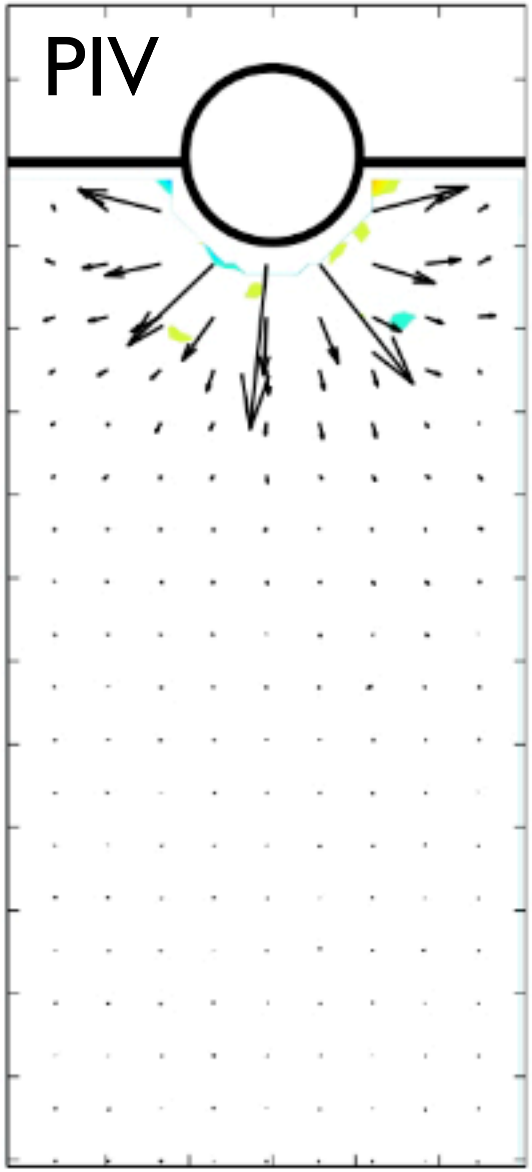
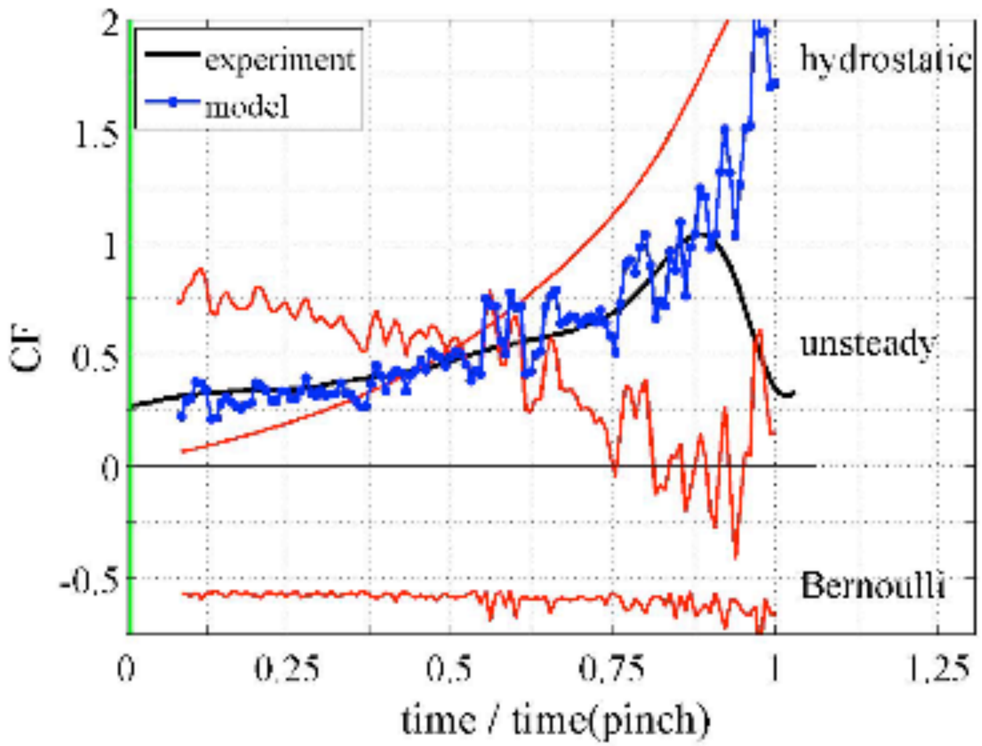


particle image velocimetry

acrylic: $m^* = 1.2$



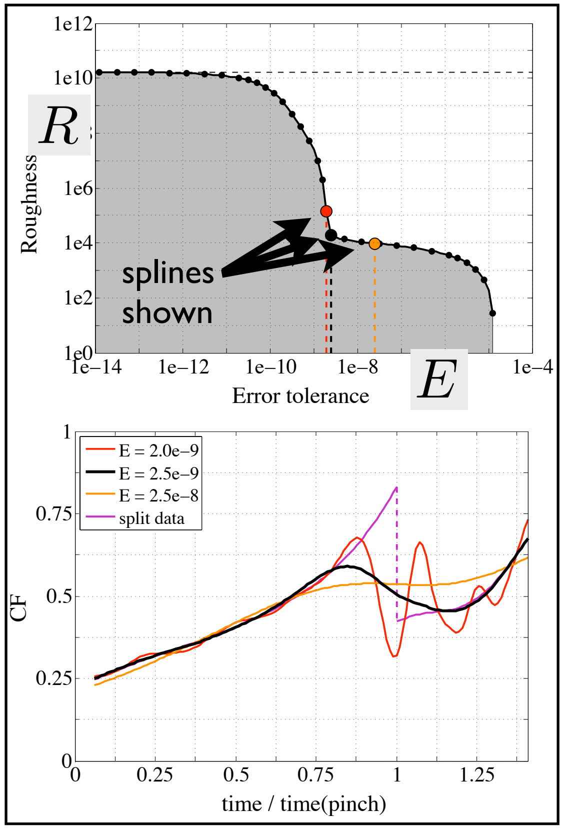
$$C_F = \frac{ma(t) + mg}{\frac{1}{2}\rho[V(t)]^2 \pi R^2}$$



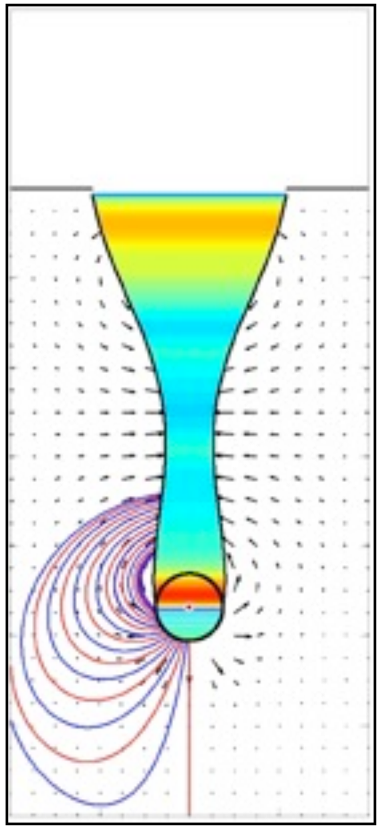
questions?

experiment

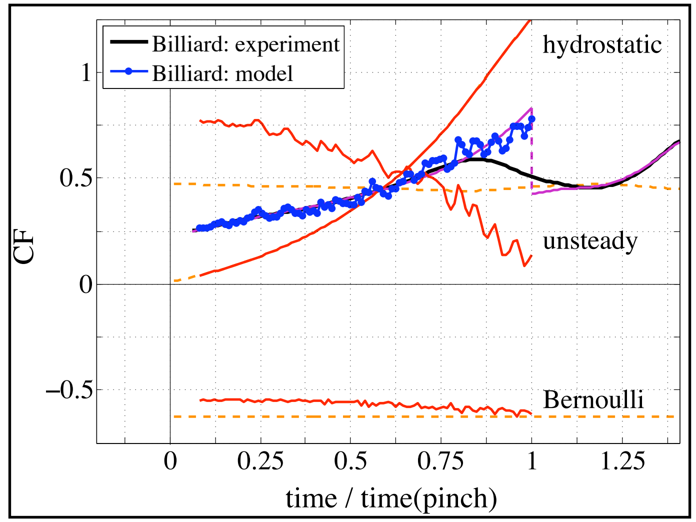
numerical method



physical model



results



Brenden Epps
bepps@mit.edu

Experimental Hydrodynamics Laboratory

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